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## Higher order systematic effect in Ptolemy's *Geographia* coordinate description of Iberia

*Keywords:* Ptolemy's *Geographia*; Ptolemy's coordinates; Ptolemaic reference system; best fitting; spatial distribution of longitude and latitude differences.

### *Summary*

Claudius Ptolemy, in his *Geographia* describes geographic sites (i.e. towns, mountain picks, river mouths, promontories and other) as points with given coordinates of spherical longitude and latitude type. These geographic coordinates are following the known Ptolemaic reference system of parallels and meridians, the origin of which are respectively close to actual Equator and close to the Canary Islands many degrees west of the today's origin at Greenwich. It is also known that though latitudes is rather well defined, considering the level of measuring accuracy at Ptolemy's times, the longitudes suffer severe shortcomings which are due to the difficulties of measurement time, which corresponds to the longitude. The longitude values given by Ptolemy are also strongly dependent upon the distance from the Canaries eastwards. In the paper, part of a broader research carried out the last years by the Cartography Group of our Faculty, we focus our interest on Ptolemy's coordinates given in *Geography* for Iberia. Storing digitally the coordinates for the area of interest (almost 520 pairs of coordinates), and snooping the data, which is a laborious process because it requires the cross-checking with the relevant coordinates given in a number of Ptolemy's *Geographia* editions (in our case there are used four), the finally accepted list is formed which is compared with their today's values. The core of the study concerns a two-dimensional spatial analysis of the field of differences, testing various transformation functions in order to determine and eliminate the systematic error pattern inherent in Ptolemy's coordinates. The result, using new "reductive" methods in the comparison analysis (e.g. the concepts of the unit sphere, of the common projective support) with all affined illustrations of the associated test, shows the pattern of coordinate differences free of systematic effects up to the 2<sup>nd</sup> order, testing also and some higher order effects in order to get a better understanding of the whole process. Finally, a field of various classes of spatial deformations of isotropic and anisotropic character, is once more, tested and visualized.

### Introduction

The interest in the geometric properties of historic maps has never been exhaustively and continuously treated by analytical means, especially in the modern era of cartography. The analytical treatment of the geometric background of early maps is an issue that today attracts the attention it deserves, as a result of the challenging perspectives opened by new digital technologies. These new technologies offer generously adequate processing tools that allow diving into the world of the geometric origin and properties of historic cartographic representations and maps.

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Previous research showed the order of magnitude of the longitude and latitude differences of Ptolemy's values from the today's counterparts both in broader and local scale (Livieratos, 2006), diving into a systematic geodetic approach on the issue for the area of actual Greece (Tsorlini, Livieratos, 2006).

The core of this study concerns a two-dimensional spatial analysis of the field of differences, testing various transformation functions in order to determine and eliminate the systematic error pattern, inherent in Ptolemy's coordinates for the biggest part of Iberian Peninsula. The result, using 'reduction' methods in the comparison analysis (e.g. the concepts of the unit sphere, of the common projective support) with all affined illustrations of the associated test, shows the pattern of coordinate differences free of systematic effects up to the second order.

### Ptolemaic reference system and coordinates

Ptolemy, in his *Geographia*, gives a list of geographic coordinates of spherical longitude and latitude of almost ten thousand of point locations, on the earth surface, as known at his times. These points are referred to geographic sites (i.e. towns, mountain picks, river mouths, promontories and other) and their geographic coordinates are following the known Ptolemaic reference system of parallels and meridians, the origin of which is respectively close to actual Equator and close to the Canary Islands almost 18 degrees west of the today's origin at Greenwich (Figure 1).

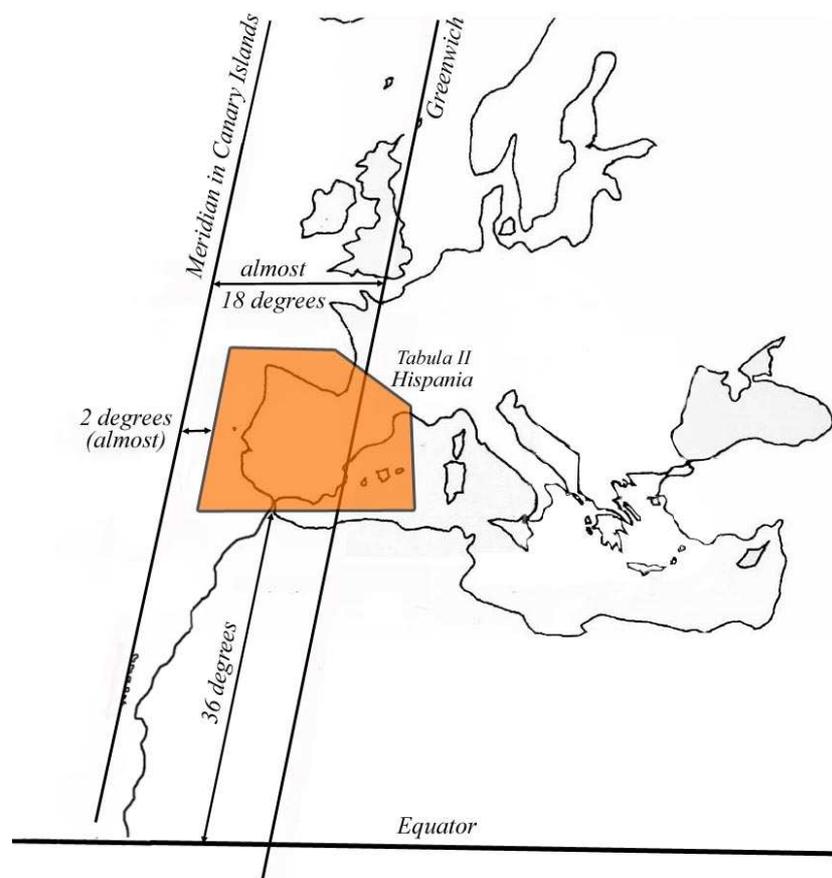


Figure 1. The origin of parallels and meridians in Ptolemy's *Geographia*.

## Coordinates in Ptolemy's *Geographia*

The world of Ptolemy is classified in Regions, since each chapter is referred to one of them, giving by this way the Atlas concept. The smaller the table is the more important and detailed the region appears to be in Ptolemy's *Geographia*, as it is obvious from the next Figure (Figure 2).

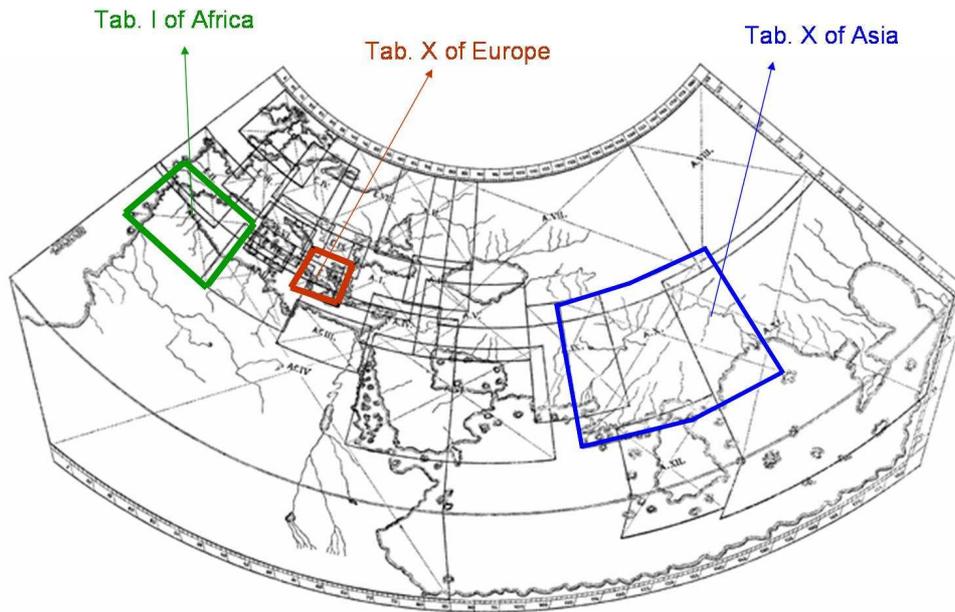


Figure 2. The 'Tabulae' in Ptolemy's *Geographia*.

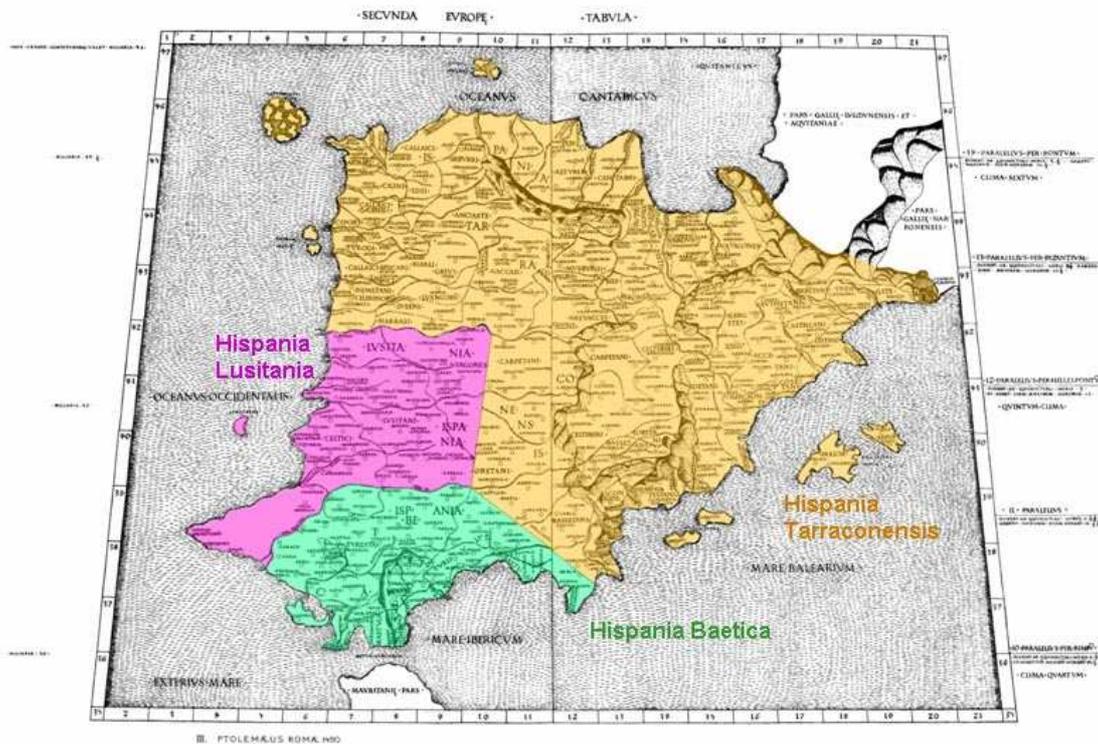


Figure 3. Table II in Ptolemy's *Geographia* and the regions of Hispania depicted on it.

In this paper, which is a part of a broader research carried out the last years by the Cartography Group in the Faculty of Surveying Engineering at Thessaloniki, we focus our interest on Ptolemy's coordinates given in *Geographia* for Hispania listed in Book II, Chapters III to V and depicted in Table II of Europe. There are three regions in Iberia, which are Hispania Baetica, Hispania Lusitania and Hispania Tarraconensis, and they are depicted in Figure 3.

In this case, we are talking about almost 520 pairs of coordinates, from which almost 470 refer to the actual territory of Spain. The editions, we use for this particular study are the following four:

- the *Vatopedion Codex* (13<sup>th</sup> -14<sup>th</sup> century),
- the *Marciana Codex* (15<sup>th</sup> century),
- the *Donnus Nicolaus Germanus* mid-15<sup>th</sup> century manuscript of Ptolemy's *Geographia* as given in Codex Ebnerianus (Stevenson 1991: 92) and
- the printed edition of *Ptolemaios, Handbuch der Geographie* by A. Stueckelberger -G. Grasshof, Basel, 2006

### Processing the Ptolemy's Coordinates

According to the procedure we follow, we first collect the coordinates of the area of interest from the different editions of Ptolemy's *Geographia* we have, we transcribe them from Byzantine writing, if it's necessary, and then, we store them digitally in a database (Table 1), having by this way the digital cataloguing of geographic coordinates.

id	Toponym	Germanus	Gerl	German	Gerl	Bem edition	Bem	Bem	Bem	Marciana Cod	Marc	Marciana Marc	Vatopedion	Vato	Vitope	Vitop
800	mouth of Ana river	4	20	37	30		4	20	37	30		4	20	37	30	
801	river Ana from east	6	20	39	0		6	20	39	0		6	20	39	0	
802	river Ana in Lusitania	9	0	38	0		9	0	38	0		9	0	38	0	
803	terminus in Balearic sea	12	0	37	15		12	0	37	15		12	0	37	15	
804	sources of river Ana	11	0	40	0		14	0	40	0		14	0	40	0	
805	Ondra estuary	4	40	37	20		4	40	37	20		4	40	37	20	
806	east mouth Bsetis river	5	20	37	0		5	20	37	0		5	20	37	0	
807	sources of river Baetis	12	0	38	30		12	0	38	30		12	0	38	30	
808	Estuary near Asta	6	0	36	45		6	0	36	45		6	0	36	45	
809	Menesthus harbor	6	0	36	20		6	0	36	20		6	0	36	20	
810	Temple of Juno (prom)	5	45	36	5		5	45	36	5		5	45	36	5	
811	mouth of Baetis river	6	10	36	10		6	10	36	10		6	10	36	10	
812	Baelonis town	6	15	36	5		6	15	36	40		6	15	36	40	
813	Menalia	6	30	36	5		6	30	36	30		6	30	36	30	
814	Transducta	6	50	36	5		6	40	36	20		6	40	36	20	
815	Barbesola town	7	15	36	10		7	15	36	10		7	15	36	10	
816	Carteis	7	30	36	10		7	30	36	10		7	30	36	10	
817	Calpe mountain (Gibraltar)	7	30	36	15		7	30	36	15		7	30	36	15	
818	mouth of Barbesola river	7	40	36	20		7	40	36	40		7	40	36	40	
819	Suel	8	0	36	55		8	0	36	55		8	0	36	55	
820	mouth of Salduba river	8	30	37	0		8	30	37	0		8	30	37	0	
821	Malaca (Malaga)	8	50	37	30		8	50	37	30		8	50	37	30	
822	Maencha	9	15	37	15		9	15	37	15		9	15	37	15	
823	Sax	9	45	37	15		9	45	37	15		9	45	37	15	
824	Selambina	10	15	37	15		10	15	37	15		10	15	37	15	
825	Projecting land (procurus)	10	50	37	5		10	50	37	5		10	50	37	5	
826	Abdera	10	45	37	10		10	45	37	10		10	45	37	10	
827	Great Harbor (Portus Magnus)	11	20	37	5		11	20	37	5		11	20	37	5	
828	Charidem promontory	11	30	36	50		11	30	36	50		11	30	36	50	
829	Bania town	11	45	37	10		11	45	37	10		11	45	37	10	
830	Sotia	9	10	38	50		9	10	38	50		9	10	38	50	

Table 1. A part of the database showing the coordinates in all the editions used for this study.

The coordinates from the four sources, are independently and mutually checked and evaluated through this database, in order to detect discrepancies in the point placement, gross errors, double values an edition may have for the same toponym, or lack of values and toponyms in some editions (Table 2).



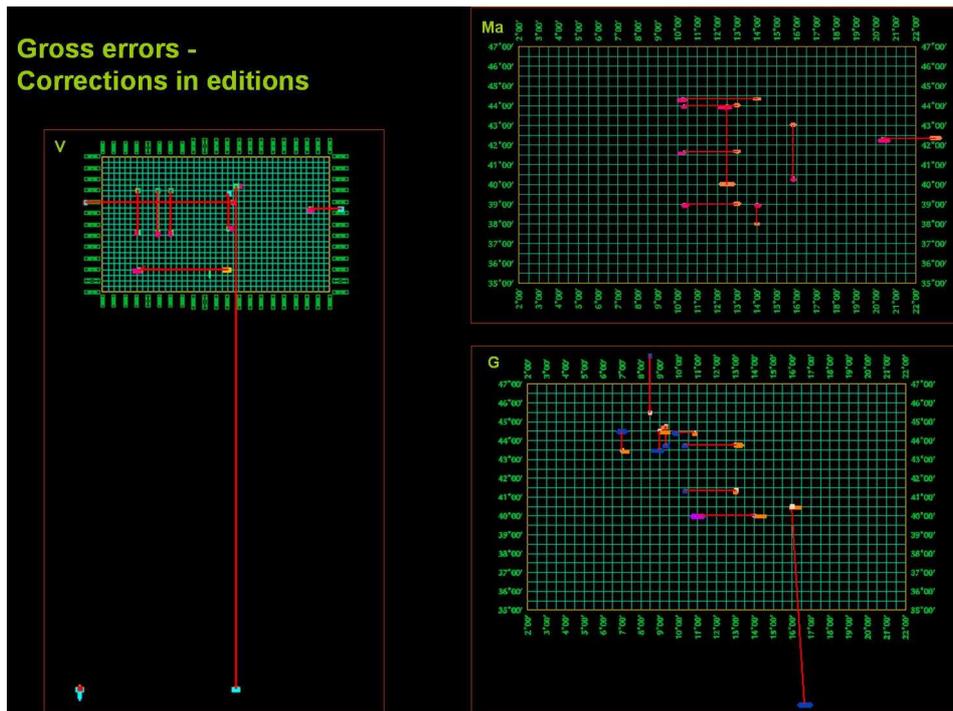


Figure 3. The visualization of gross errors and their correction in each edition.

The projection of toponyms to a map for the editions used in this study, after their correction from gross errors, is shown in the images below (Figure 4). The initial letter in the right corner of each map indicates the edition of Ptolemy's *Geographia*, while the different colors of the points are referred to the different regions as described in *Geographia*.

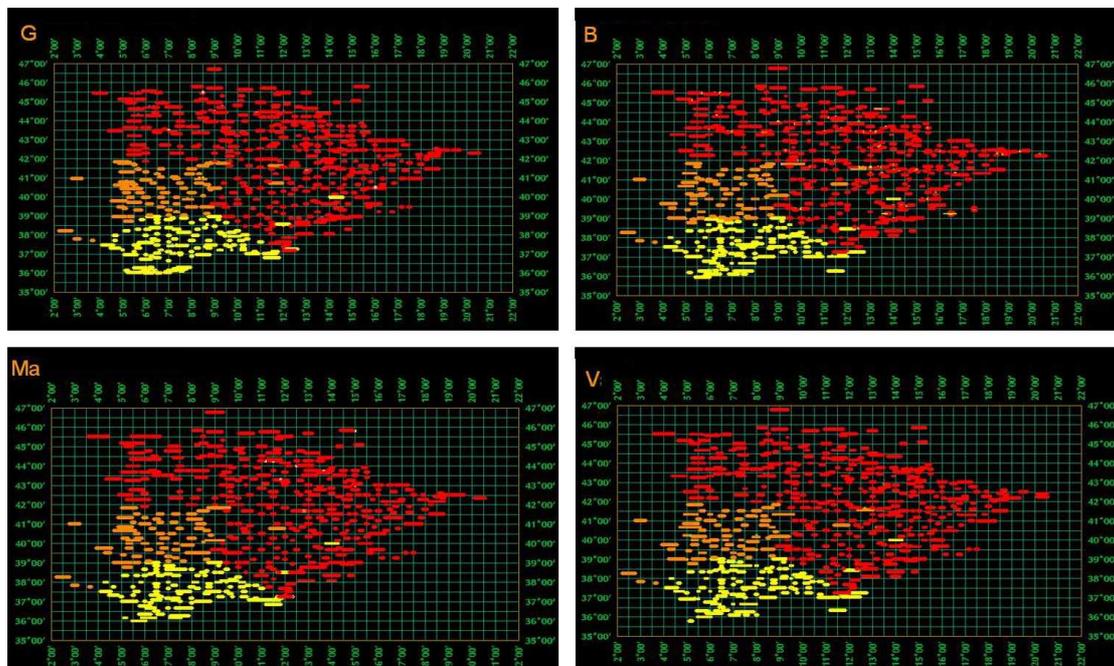


Figure 4. Coordinate plotting in geographic projection according to the snooped list of Ptolemy's coordinates for Iberia. The initial letters on the four images indicate the edition of Ptolemy's *Geographia* (G: Germanus' edition, B: Ptolemaios, Handbuch der Geographie, Basel, 2006, Ma: Marciana Codex and V: Vatopedion Codex).

Having projected the coordinates on the map, we compare and analyze them using geodetic methods, so as to try to conclude to an accepted list without gross errors, double values or records or other apparently erroneous discrepancies in point placement.

### The identification of ancient toponyms with their modern counterparts

Another important procedure in this study is the identification of ancient toponyms with their modern counterparts. Having concluded to an 'accepted' list of coordinates without gross errors, double values or records or other apparently erroneous discrepancies in point placement, we start the comparison of Ptolemy's coordinates with their today's counterparts. In order to perform such a comparison and to identify the coincidence of places in Ptolemy's era with their today's counterparts, we had to compare the toponyms of each area of Ptolemy's Hispania with the toponyms of the corresponding area of actual Spain and Portugal, confirming at the end the coincidence of the with certainty known points in both cases, based mainly on old maps and relevant references in historical and archeological sources.

In this study, the whole inquiry of maps, old and modern, and of the other historical data, collected and used for the identification of ancient toponyms with modern, is based on internet. The criteria used for the selection of the maps are mainly their resolution - the bigger resolution the map has, the better and easier it can be read after its fitting to the graticule, plotted for the area of interest - and the existence of geographic graticule on the map, which helps maps' fitting to the 'plotted' graticule. With regard to old maps, it is also important to check three more things before the selection of an old map. These are the prime meridian, the toponyms on the map and the coastline.

In ancient times, there were various conventions for the prime meridian, something that influences straightly the old maps. Ptolemy used in his *Geographia*, as prime meridian, that in Canary Islands. The modern prime meridian is passing through Greenwich. For this reason, it is important to select a map with the prime meridian in Greenwich, so that the control points used in the best fitting process can be easily found.

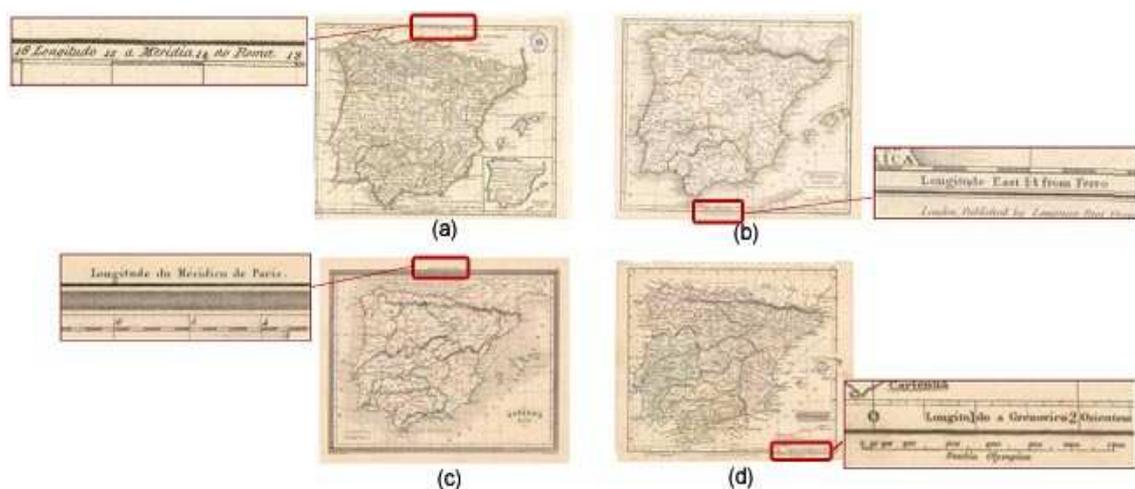


Figure 5. Historical maps with different prime meridian  
(a) Hispania antiqua, A. Dell, B. Borghi, Firenze, 1819, (b) Hispania antiqua, Sidney Hall, 1830, (c) Espagne ancienne / Dressée par A. Villemin ; Lale direct. et scrip, 1800, (d) Hispania et Insule, John Arrowsmith, 1840

Regarding the toponyms, it is important to find maps with ancient toponyms together with the modern coastline, because in that way the modern places where the ancient toponyms are located, can be more easily detected. For this reason, it is important for us to use maps, where the coastline of Iberian Peninsula and of the islands around it, is similar to the modern one, so that it can be fitted exactly to the modern map. The next figures show examples of these two last cases (Figure 6-7).

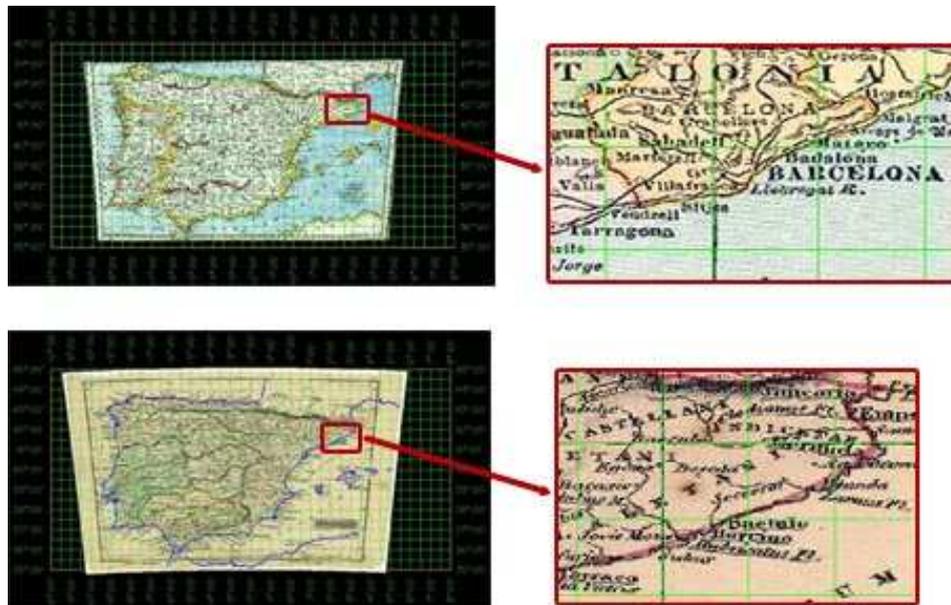


Figure 6. Maps of Spain and Portugal with different toponyms.

The first map (*A Map of Spain & Portugal*, published in Robert Wilkinson's *General Atlas*, circa 1794) has modern toponyms and the other (*Hispania et Insule*, A. & S. Arrowsmith, 1829) ancient.

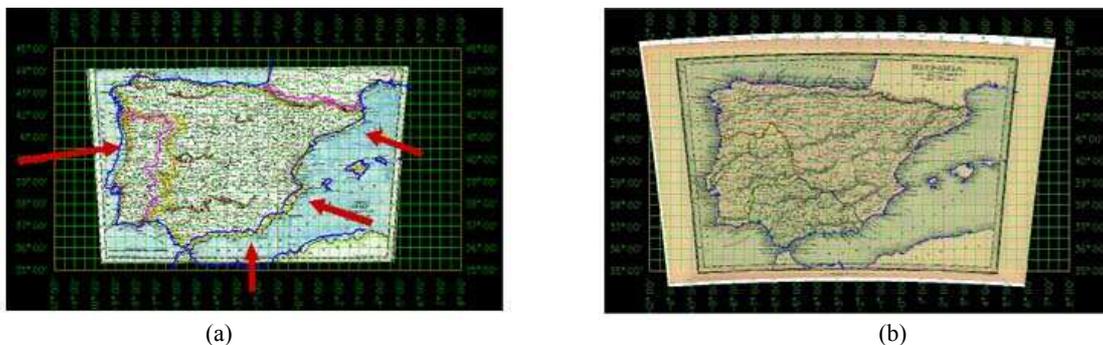


Figure 7. Ancient maps best fitted to the modern. The coastline of Iberian Peninsula in the first map doesn't fit well to the modern, while in the second, the coastline fits exactly ((a): *Map of Spain and Portugal*, C.S. Hammond & Co., N.Y., 1921, (b): *Hispania*, Alex Findley, published by T.Tegg, London, 1830)

Based on these criteria, at first, modern maps and then old maps are selected and fitted with the best possible way to the 'plotted' graticule, using as control points, the common nodes of the graticule. The procedure followed in this case is shown in the figure below (Figure 8).

The old maps, selected and fitted to the modern map in order to help to the identification of ancient toponyms with modern are:

- a. *Hispania antiqua*, Sidney Hall, 1830 (Figure 9d)

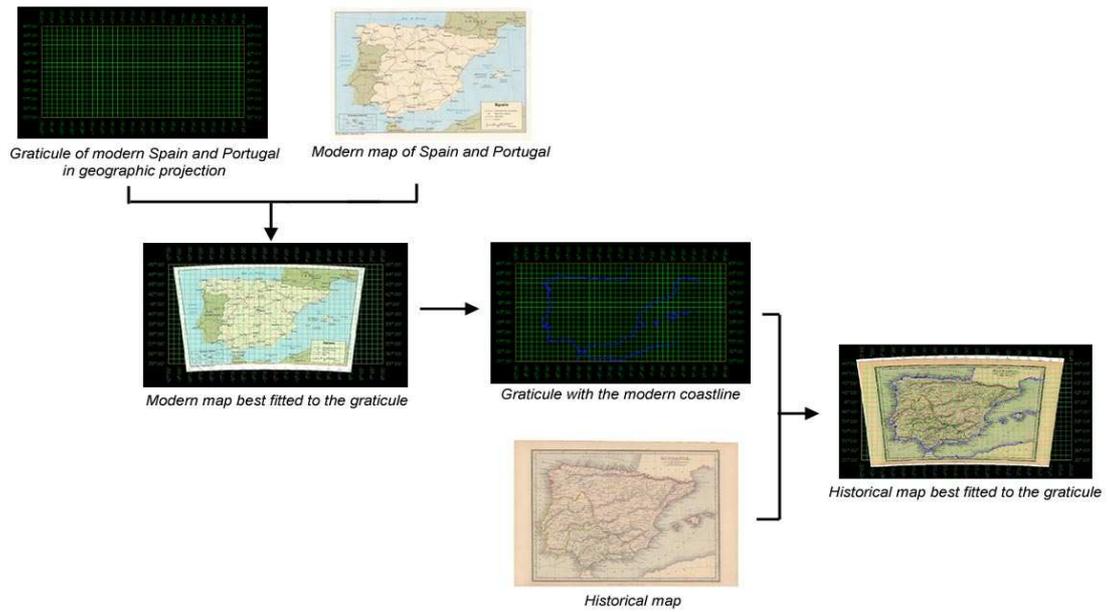


Figure 8. The best fitting of modern and old maps to the graticule plotted in the area of actual Spain and Portugal

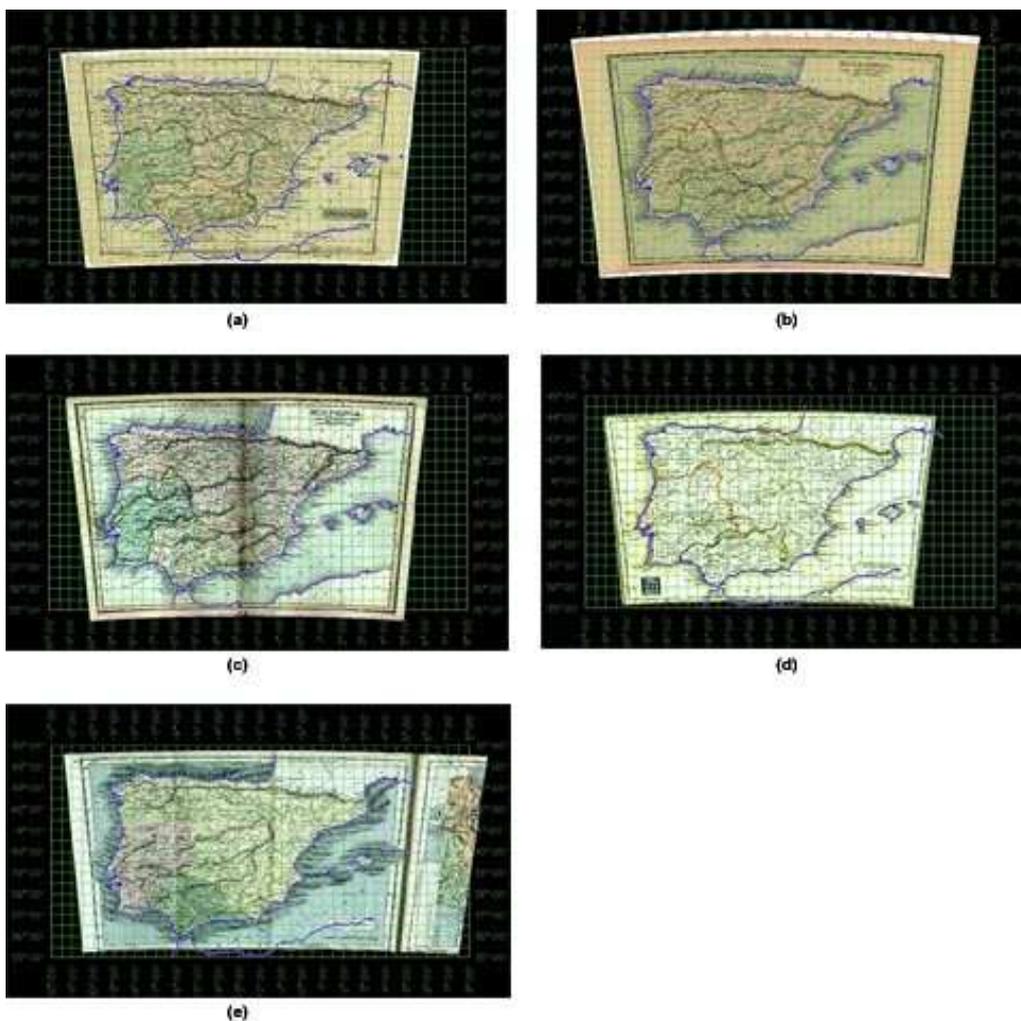


Figure 9. Old maps (a) Hispania et Insule, John Arrowsmith, 1840, (b) Hispania, Alex Findley, London, 1830, (c) Map of Spain, Classical Atlas of Ancient Geography, A. G. Findlay, New York, 1849 (d) Hispania antiqua S. Hall, 1830 (e) Hispania, Atlas Of Ancient And Classical Geography, J. M. Dent And Sons, 1912

- b. Hispania, Alex Findley, published by T. Tegg, London, 1830 (Figure 9b)
- c. Hispania et Insule, John Arrowsmith, 1840 (Figure 9a)
- d. Map of Spain from A Classical Atlas of Ancient Geography by Alexander G. Findlay. New York: Harper and Brothers 1849. (Figure 9c)
- e. Hispania-Spain, Atlas Of Ancient And Classical Geography, J. M. Dent And Sons, 1912 (Figure 9e)

Having compared the toponyms, the ancient with modern, we concluded to have almost 300 identified points, without counting on them the mountains and some physical borders, Ptolemy included in his Geographia. Most of these points will be used as control points in best fitting Ptolemy's map for Hispania to the modern map of Spain and Portugal. In the next map (Figure 10), we can see on a modern map, the places, where most of Ptolemy's toponyms in Iberian Peninsula, are detected according to historical and other sources.

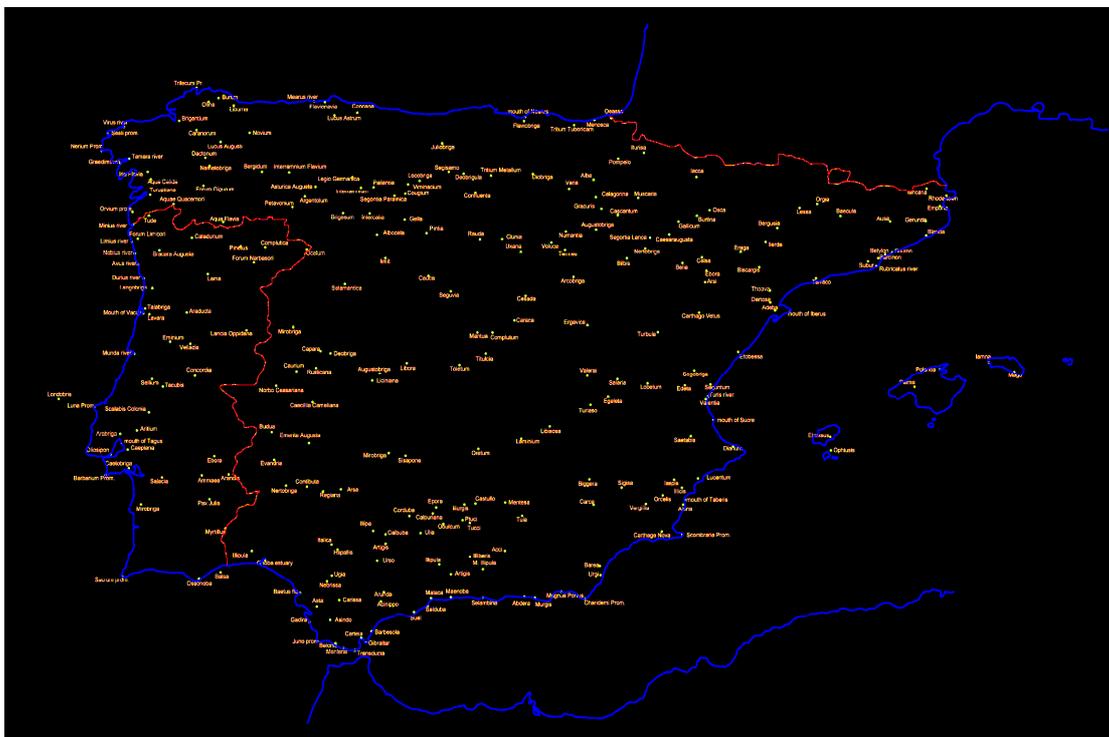


Figure 10. Ptolemy's toponyms of Hispania depicted on a modern map

### Best fitting of Ptolemy's representation to a modern map

The points, we have mentioned before, have great importance to the continuity of this work because a set of them, properly distributed to the overall map space, is selected and brought into one to one correspondence with the actual coordinates of the same set of points in the modern map, after choosing a transformation system, in this case a 2<sup>nd</sup> order polynomial transformation, involving a projection and an earth's model. The result of the best fitting of Ptolemy's coordinates to the modern counterparts is shown in Figure 11. The Ptolemy representation is georeferenced to actual geographic coordinates using almost 270 control points properly distributed in the area of Spain and Portugal. Ptolemy's graticule, extended from 2° to 21° in longitude and from 36° to 47° in latitude, contrary to the geographic graticule of the modern map, which extended from -10° to 5°

in longitude and from 36° to 44° in latitude. In the resulting map (Figure 11), Ptolemy's map of coordinates is transformed into the actual coordinates and the deformation appeared in Ptolemy's graticule is obvious.

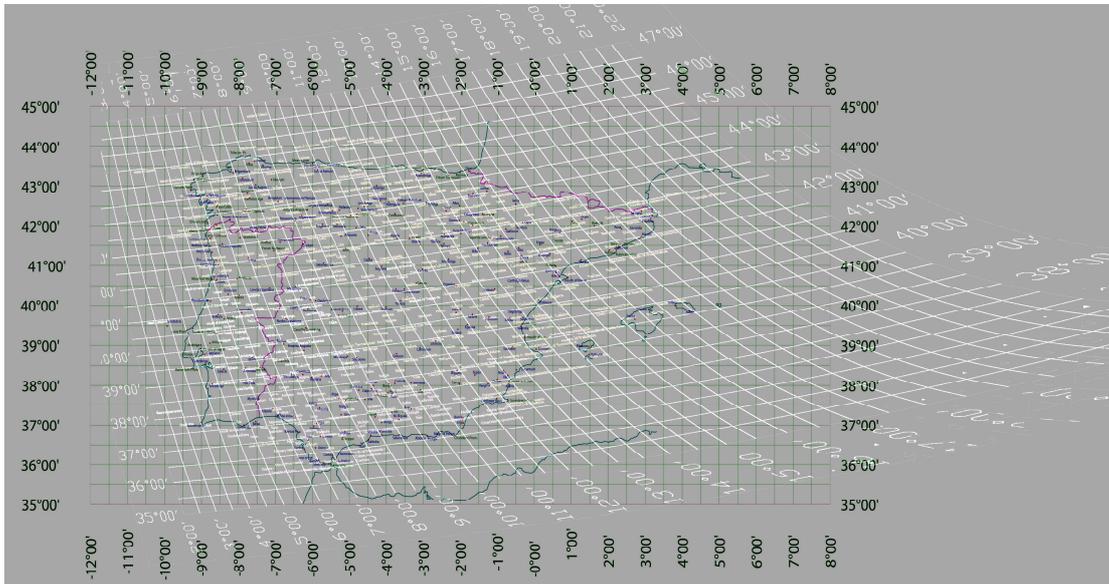


Figure 11. Second order polynomial best fitting of Ptolemy's representation of Hispania to a modern relevant map

### The spatial distribution of differences in longitude and latitude

Using the best fitting of Ptolemy's representation to the modern map, we study also, the spatial distribution of the differences in longitude and latitude induced after the comparison of Ptolemy's coordinates with their actual values. In the next two figures, which depict the distribution of the differences in both cases (Figure 12 and 13), it is obvious that the distribution is not the same.

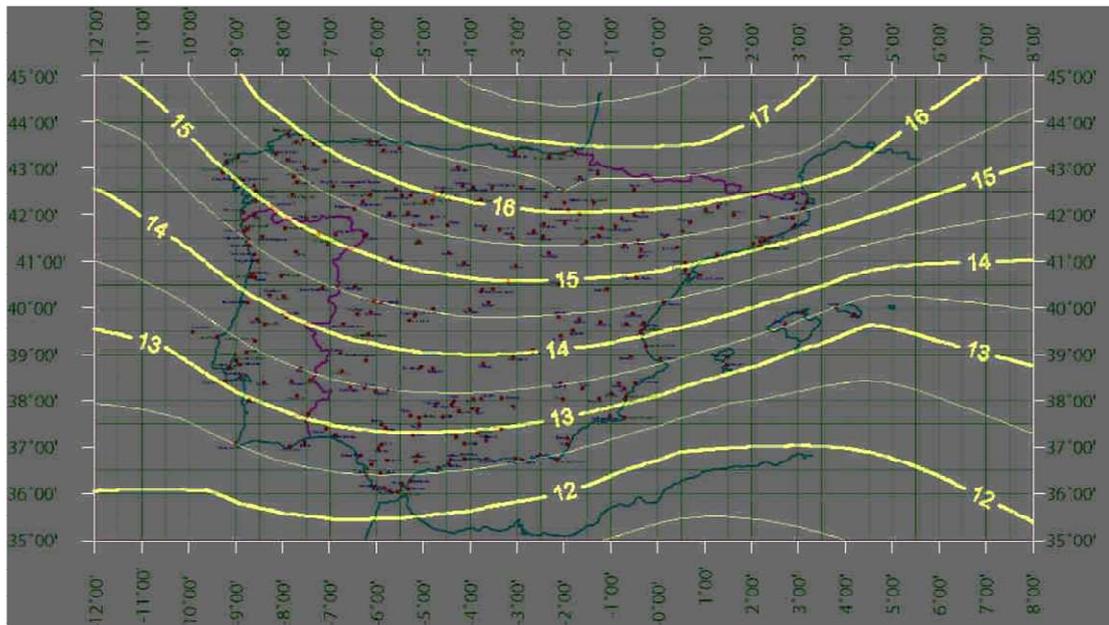


Figure 12. The isolines of longitude differences, in degrees, between Ptolemy's values and their actual counterparts

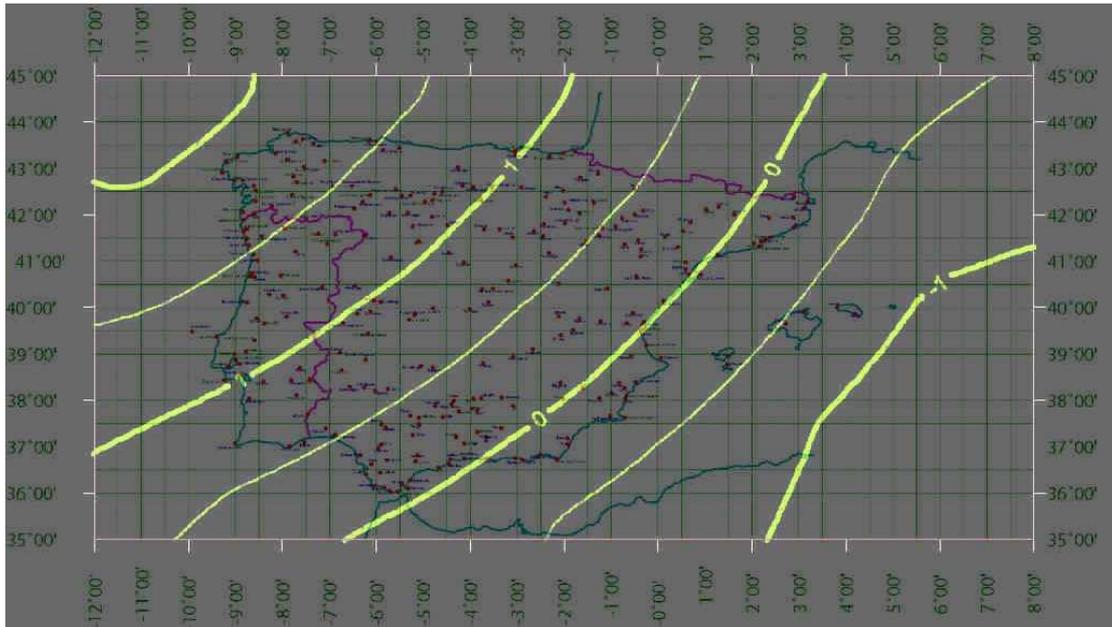


Figure 13. The isolines of latitude differences, in degrees, between Ptolemy's values and their actual counterparts

As we can see above, the longitude differences vary from 12° at Gibraltar, the south side of Iberian Peninsula to 17° at the north side, whereas the latitude differences are of much smaller magnitude than those of longitude and vary from -0.5° east, in Balearic Islands to almost 2° at northwest. These differences can be easily explained by the fact that though latitudes are rather well defined, considering the level of measuring accuracy at Ptolemy's times, the longitudes suffered severe shortcomings which are due to the difficulties in measuring the time, which corresponds directly to longitude. Moreover, the longitude values given by Ptolemy are strongly dependent upon the distance from the Canaries eastwards.

### Concluding remarks

The advances of digital computational and visualization technologies (informatics and infographics) which are massively available today allowing new approaches and techniques in studying this extraordinary document of our cartographic heritage as it is the Ptolemy's *Geographia*. The transformation of early maps into digital form and their comparison with modern maps using new processing methods and technologies is of great importance for the study of the geometric properties of early cartographic documents. Best fitting techniques are appropriate in order to compare early cartographic representations with their modern counterparts.

This study particularly, as well as previous research, both in broader and local scale, showed the order of magnitude of the longitude and latitude differences of Ptolemy's values from the today's counterparts. The result of the two-dimensional spatial analysis of the field of differences in Ptolemy's coordinates shows the pattern of coordinate differences free of systematic effects up to the 2<sup>nd</sup> order. This work is extended by testing also and some higher order effects in order to get a better understanding of the whole process.

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#### Modern maps of Iberia from internet

[www.map-of-spain.co.uk](http://www.map-of-spain.co.uk)

[www.gomadrid.com/madrid-maps.html](http://www.gomadrid.com/madrid-maps.html)

[www.fiestavillas.co.uk](http://www.fiestavillas.co.uk)

[www.lib.texas.edu](http://www.lib.texas.edu)

[www.spain-calling.com](http://www.spain-calling.com)

[www.sacred-destinations.com/spain/spain-maps.html](http://www.sacred-destinations.com/spain/spain-maps.html)

[http://www.go.hrw.com/atlas/norm\\_hm/spain.htm](http://www.go.hrw.com/atlas/norm_hm/spain.htm)

[http://www.shunya.net/Pictures/Spain/Spain\\_map.gif](http://www.shunya.net/Pictures/Spain/Spain_map.gif)